SIEMENS STEM DAY | Science Technology Engineering Math

GRADES 9–12

SIEMENS STEM DAY ACTIVITY

A BALL IN MOTION

OBJECTIVES

Students will be able to:

- **Understand** the mathematical relationship between the parabolic path of a ball in motion and quadratic functions
- **Gather,** calculate and analyze data related to a ball in motion
- **Apply** the quadratic formula to find the velocity and maximum height of a ball in motion
- **Graph** and **label** a real-world quadratic function

THIS LESSON FOCUSES ON

Engineering Design Cycle

- Defining the Problem
- Communicating Results

21st Century Skills

- Collaboration
- Critical Thinking

OVERVIEW

Students will investigate the relationship between quadratic functions and the parabolic path travelled by a ball in motion. By conducting an experiment, students will gather and analyze data to understand the mathematical relationships that exist along the path of a ball in flight.

STEM incorporates Science, Technology, Engineering, and Mathematics to focus on real-world issues and problems guided by the engineering design process. This type of instruction supports students in developing critical thinking, collaboration, reasoning, and creative skills to be competitive in the 21st-century workforce.

Each Siemens STEM Day classroom activity highlights one or more components of the engineering design cycle and an essential 21st-century skill.

MATERIALS

- **A Ball in Motion Activity** Handout—one per person
- **A Ball in Motion Activity** Handout KEY—one copy
- Tennis balls- one per partner set
- Meter sticks- one per partner set
- Stopwatches- one per partner set

HAVE YOU EVER WONDERED . . .

Are there mathematical connections in sports or some video games? If so, how?

MAKE CONNECTIONS!

How does this connect to students?	How does this connect to careers?	How does this connect to our world?
Many students are involved in sports or even enjoy watching sports. By understanding the trajectory of the path of a ball in flight, students could possibly improve their skills, as well as, better understand the world in which they live. Additionally, many students take a **physics** class and this activity demonstrations the connections that exist between mathematics and the physical world.	A **video game programmer** is responsible for developing and writing code that assists in the movement of game characters or items. For instance, the developers of the game Angry Birds program the game so that the users actually interact with properties and applications of projectile motion. By changing the slingshot height, angle, and the force used, the gamer begins to identify patterns that occur as each variable is changed. In addition to many other tasks, a **NASA** or **rocket engineer** is responsible for trajectory design and analysis. There are many variables that impact the path of an object in motion. The applications of quadratics and vectors are critical aspects of successful launches.	**Gravity** is responsible for pulling all objects to Earth's center. The concept of inertia states that objects in motion will remain in motion until a force acts upon it. When a ball is thrown into the air, the force of gravity is responsible for creating a parabolic path of trajectory. Without the presence of air (in a vacuum state), all objects would fall to Earth at a constant rate of acceleration, regardless of their masses. This constant rate of gravitational acceleration is 9.8 m/s^2. However, due to the presence of **air resistance**, different objects fall faster or slower depending upon a multitude of variables.

BLUEPRINT FOR DISCOVERY

1. To kick off the Ball in Motion activity, play the following video for the students. This will demonstrate the connection that exists between the path of a ball in motion and the mathematical application of quadratics. https://fast.wistia.net/embed/iframe/o6t4k1e83x
2. Tell students that they are going to partner up and conduct a lab activity. Pass out **A Ball in Motion Activity Handout**, one to each student. This activity is best done outside in an open area, if that is not possible secure a large space indoors.
3. Explain to the students that this is an activity in which they will throw a tennis ball and apply mathematical skills and understanding to determine the velocity (speed in a given direction) and maximum height of the ball. It is best for students to have some prior knowledge with solving and understanding quadratic functions.

4. Pass out one tennis ball, one meter stick and one stopwatch to each partner set. Alternately, students can use the timer on their phone or an electronic device.
5. Remind the students to closely follow the directions and ask clarifying questions as they arise. Emphasize to the students that they will use meters for measuring distance and seconds for measuring time. Depending upon time constraints, decide whether you will instruct each partner set to collect ONLY one set of data or if you'd like partner sets to assist one another in collecting their OWN set of data. In other words, will each partner set have one thrower or will each student get a chance to throw the ball result in each person having their own data.
6. Once students have completed the ball throwing portion of the **Ball in Motion** activity, allow them some additional time to complete the calculations and answer the questions.
7. When all partner sets are ready, regroup as a class and discuss the students' results. Draw a data table (similar to the one below) on the board to collect the class data.

Group Number	Release Height (m)	Average Time (s)	Initial Velocity (m/s)	Maximum Height of the Ball (m)

These can be challenging concepts for the students to fully grasp. Therefore, be aware of inaccurate or unreasonable results. Please note that an answer key has been provided that includes an example of reasonable responses and calculations.

8. To conclude this lesson, ask students if they can think of any video games that utilize these mathematical and physics-related concepts. Potential answers may include sports related video games, Angry Birds, Paper Toss, etc.

TAKE ACTION!

- Students can apply their graphing skills and represent this data visually.
- Also, students can create and solve their own related problem(s) and then switch with their peers to see if they arrive at the same solution(s).

NATIONAL STANDARDS

Standards for Technology Literacy	Standard 13: Students will develop Abilities for a Technological World. This includes becoming able to assess the impact of products and systems. Standard 16: Students will develop an understanding of The Designed World. This includes selecting and using energy and power technologies.
Next Generation Science Standards	HS-PS2-4: Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.
Next Generation Science Standards Science and Engineering Practices	SEP3: Planning and Carrying Out Investigations SEP4: Analyzing and Interpreting Data SEP5: Using Mathematics and Computational Thinking
Common Core Math State Standards	CCSS.MATH.CONTENT.HSA.REI.B.4: Solve quadratic equations in one variable. CCSS.MATH.CONTENT.HSA.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

A BALL IN MOTION ACTIVITY

Work with a partner to complete the following activity. The materials you will need include a meter stick, a stopwatch, and a tennis ball.

1. Outside or in a large space, spread out and throw a tennis ball back and forth a few times with your partner.
2. Try throwing the tennis ball vertically up at different release heights. What do you observe about the relationship between the release height and the distance the ball travels?
3. A ball moves in the shape of a parabola which is a type of mathematical curve created by a quadratic equation. Projectile motion involves objects that are dropped, thrown up vertically or thrown straight down and can be modeled by a quadratic function.

   To solve for the height of vertical motion:

   The following formula compares the height of an object with the time in flight:

   $f(t) = -\frac{1}{2} gt^2 + vt + h$

   g = the speed of gravity = 9.8 m/s
   v = initial velocity
   h = initial height
   t = time in seconds

   This formula can be simplified to $f(t) = -\frac{1}{2}(9.8)\,t^2 + vt + h$
   $f(t) = -4.9\,t^2 + vt + h$
4. Have the person throwing the ball stand with his/her arm extended high into the air. Measure this release height of the ball.

   Release height: __________ meters
5. Time the person throwing the ball vertically straight up from the time of release to when the ball hits the ground. Repeat this process 6 times and then find the average time in seconds.

Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Average time(s)

   Average time: __________ seconds

6. Calculate the initial velocity (v) of the ball at a initial time of 0 seconds using the formula: $f(t) = -4.9\,t^2 + vt + h$ and substituting in the information above.

   $0 = -4.9\,(\text{average time})^2 + v(\text{average time}) + (\text{release height})$

   The initial velocity of the ball is __________ meters per second.

7. Now write the equation of the function using the information that you gathered and calculated. Substitute the calculated initial velocity for v and the initial height for h, leaving the remainder of the formula as a function of time (t).

   $f(t) = - 4.9\, t^2 + vt + h$

8. Calculate the maximum height of the ball:

   Recall that this is a quadratic equation. In order to solve for the maximum height, you will solve for the vertex point using $x = -b/2a$. For this problem, the formula can be rewritten in the context of the problem as: $t = -v/\ 2(-4.9)$

   Using the result for t (found above), you can solve for the height at this given time h(t).

   f(t) = - 4.9 t2 + vt + h

   f(t) = - 4.9 (time at max height)2 + velocity(time at max height) + (initial height)

   The maximum height of the ball is __________ meters.

9. Consolidate your data into the table below.

Group Number	Release Height (m)	Average Time (s)	Initial Velocity (m/s)	Maximum Height of the Ball (m)

A BALL IN MOTION ACTIVITY | SAMPLE ANSWER KEY

Work with a partner to complete the following activity. The materials you will need include a meter stick, a stopwatch, and a tennis ball.

1. Outside or in a large space, spread out and throw a tennis ball back and forth a few times with your partner.
2. Try throwing the tennis ball vertically up at different release heights. What do you observe about the relationship between the release height and the distance the ball travels?
3. A ball moves in the shape of a parabola which is a type of mathematical curve created by a quadratic equation. Projectile motion involves objects that are dropped, thrown up vertically or thrown straight down and can be modeled by a quadratic function.

   To solve for the height of vertical motion:

   The following formula compares the height of an object with the time in flight:

   $f(t) = -\frac{1}{2}gt^2 + vt + h$

   g = the speed of gravity = 9.8 m/s
   v = initial velocity
   h = initial height
   t = time in seconds

   This formula can be simplified to $f(t) = -\frac{1}{2}(9.8)t^2 + vt + h$
   $f(t) = -4.9t^2 + vt + h$

4. Have the person throwing the ball stand with his/her arm extended high into the air. Measure this release height of the ball.

   Release height: 2.3 meters (A 5-foot student with a reach length of 31 inches is a total of 91 inches. In meters, this is a distance of 2.3 meters.)

5. Time the person throwing the ball vertically straight up from the time of release to when the ball hits the ground. Repeat this process 6 times and then find the average time in seconds.

Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Average time (s)
1.75	1.86	2.01	1.90	1.79	1.98	1.87

   Average time: 1.87 seconds

6. Calculate the initial velocity (v) of the ball at a initial time of 0 seconds using the formula: $f(t) = -4.9t^2 + vt + h$ and substituting in the information above.

   $0 = -4.9(\text{average time})^2 + v(\text{average time}) + (\text{release height})$

   $0 = -4.9(1.87)^2 + v(1.87) + 2.3$
   $0 = -17.13 + 1.87v + 2.3$
   $0 = -14.83 + 1.87v$
   $14.83 = 1.87v$
   $7.9 = v$

   The initial velocity of the ball is 7.9 meters per second.

7. Now write the equation of the function using the information that you gathered and calculated. Substitute the calculated initial velocity for v and the initial height for h, leaving the remainder of the formula as a function of time (t).

f(t) = - 4.9 t2 + vt + h
f(t) = - 4.9 t2 + 7.9t + 2.3

8. Calculate the maximum height of the ball:

Recall that this is a quadratic equation. In order to solve for the maximum height, you will solve for the vertex point using x = -b/2a. For this problem, the formula can be rewritten in the context of the problem as: t = -v/ 2(-4.9)

t = -7.9 / 2(-4.9)
t = -7.9 / -9.8
t = 0.8 seconds

Using the result for t (found above), you can solve for the height at this given time h(t).

f(t) = - 4.9 t2 + vt + h

f(t) = - 4.9 (time at max height)^2 + velocity(time at max height) + (initial height)

f(t) = - 4.9(0.8)2 + 7.9(0.8) + 2.3
f(t) = - 3.136 + 6.32 + 2.3
f(t) = 5.48 meters

The maximum height of the ball is 5.48 meters.

9. Consolidate your data into the table below.

Group Number	**Release Height (m)**	**Average Time (s)**	**Initial Velocity (m/s)**	**Maximum Height of the Ball (m)**
	2.3	1.87	7.9	5.48