SIEMENS STEM DAY | Science Technology Engineering Math

GRADES 9–12

SIEMENS STEM DAY ACTIVITY

ALL GEARED UP!

OBJECTIVES

Students will be able to:

- Understand the basic functioning of how gears work.
- Identify the reciprocal relationship that exists between the teeth ratio and gear/speed ratio of gears.
- Demonstrate an understanding of the trigonometric properties related to arch length, gear speed, and the rotation of different gear sizes.

THIS LESSON FOCUSES ON

Engineering Design Cycle

- Defining the Problem
- Designing Solutions

21st Century Skills

- Collaboration
- Communication
- Critical Thinking

OVERVIEW

Students will learn about gears, how they work, and differences in gear size as well as develop an understanding of angular speed. A variety of problems will require students to apply this knowledge and analyze a variety of gear related situations by applying arc length and other trigonometric functions to determine degrees of rotation.

STEM incorporates Science, Technology, Engineering, and Mathematics to focus on real-world issues and problems guided by the engineering design process. This type of instruction supports students in developing critical thinking, collaboration, reasoning, and creative skills to be competitive in the 21st-century workforce.

Each Siemens STEM Day classroom activity highlights one or more components of the engineering design cycle and an essential 21st-century skill.

MATERIALS

- **Station A: How to Calculate Speed Ratio** Handout, one per student
- **Station B: How Bicycle Gears Work** Handout, one per student
- **Station C: Calculating Arc Length** Handout, one per student
- Computers with internet access
- Calculators

HAVE YOU EVER WONDERED . . .

How do the gears on your bicycle work?

MAKE CONNECTIONS!

How does this connect to students?

Many children ride their bikes daily, if not weekly, weather permitting. It is quite amazing that, as you pedal, you are propelling your bike forward. What is happening when the gear is switched? Why does pedaling become easier or harder depending upon which gear is being used? Not only are there scientific connections, but also several mathematical connections that help explain how gears function.

How does this connect to careers?

A **mechanical engineer** is combines physics, mathematical principles, and material science to design, build, and maintain a multitude of mechanical systems. This type of engineering focuses on machinery and gears often play a crucial role in the proper functioning of machines. Understanding the functionality of various types of gears is a necessity to be a mechanical engineer.

How does this connect to our world?

Gears are considered to be a simple machine, as they propel circular motion, acting as a sort of lever. Gears are used in countless machines, making the workload easier and more efficient.

Circular motion is measured as speed in **rpm** (revolutions per minute). RPM can be converted into angular velocity in degrees per second. This is done by multiplying the rpm by 6 (since one revolution is 360° and there are 60 sections per minute). For instance, if a gear rotated 120 rpm, then the angular velocity is 720° per second. Likewise, if the conversion needs to go from angular velocity to rpm, the angular velocity is multiplied by 1/6.

BLUEPRINT FOR DISCOVERY

1. Hook students' interest by asking them what first comes to mind when they hear the word gear. Students often first make a connection to the gears on their bicycle. Ask students if they can define gear. A gear is a toothed wheel that engages another tooth mechanism, such as another gear or gear shaft. Next, probe further to see if students understand the purpose of gears or why they are used. Some students might be able to point out that gears increase or decrease the speed of rotation. Gears are also used to change the amount of force or torque needed to make something move.
2. There are many different types of gears such as spur, helical, beveled, or even worm gears, just to name a few. However, this lesson is going to focus on understanding some of the mathematical applications related to gears- gear ratio, gear speed, arc length, and degrees of rotation.
3. Explain to the students that they are going to engage in 3 different gear-related station activities. Each station (A-C) will focus on understanding different gear calculations with an end goal of a thorough understanding of how gears work.

4. Here is a description for each of the stations.
    - Station A: How to Calculate Speed Ratio
      Students will learn about the reciprocal relationship between the teeth ratio and gear/speed ratio of gears.
    - Station B: How Bicycle Gears Work
      Students will understand how bicycle gears function as well as the correlation that exists between gear sizes.
    - Station C: Calculating Arc Length
      Students will learn how to use trigonometric applications to solve for the arch length and corresponding degrees of rotation of two gears.
5. When deciding on the order that the groups of students will progress through the stations, it is important to note that the stations progress in difficulty level from Station A to C. Some students might benefit from being in this first group rotation A-C, so that the level of difficulty is sequential for them. Other students might be able to more easily handle the harder concepts first without the easier foundational activities. Station A begins with the mathematical application of ratios and proportions, and the stations conclude with trigonometric applications being used in Station C.
    - Optionally, you may decide to not use stations and as a class complete Activity A, B, then C sequentially.
6. To conclude this lesson, ask the "So What?" question. Encourage students to share with the class their takeaways from the lesson. Discuss "Ah Ha!" moments that took place as they participated in today's activities.

TAKE ACTION!

- Students can apply their learning of gears and make a children's picture book by visiting: https://bookcreator.com/. Use this site to create a free book that can then be digitally shared. The book can be "geared" for either elementary or middle school age children.

NATIONAL STANDARDS

Standards for Technology Literacy	Standard 8: Students will develop an understanding of Design. This includes knowing about the attributes of design. Standard 13: Students will develop Abilities for a Technological World. This includes becoming able to assess the impact of products and systems. Standard 19: Students will develop an understanding of The Designed World. This includes selecting and using Manufacturing technologies.
Next Generation Science Standards	HS-PS2-1: Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. SEP5: Using Mathematics and Computational Thinking
Common Core Math State Standards	CCSS.MATH.CONTENT.HSG.C.B.5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

ADDITIVE VS. SUBTRACTIVE MANUFACTURING

Station A: How to Calculate Speed Ratio

A gear consists of toothed wheels attached to shafts. It creates a mechanical advantage in a range of applications; for example, a cyclist uses gears to intensify the power output of his pushing on the pedals. Gears have many properties, one of which is the speed ratio, often known as gear ratio. This is the ratio of the turning speed of the input gear to that of the output gear; in other words, how many times the input gear has to revolve to make the output gear revolve once.

Speed Ratio Definition

A gear train consists of more than one gear connected to each other, and their teeth interlock. When a machine has two gears of different sizes, the smaller gear turns faster than the larger gear. When the first gear (the driver or input gear) turns, the second gear (the driven or output gear) turns in response. The difference between the speeds of the two gears is called the speed ratio or gear ratio.

Speed Ratio Calculation

The ratio is determined by the number of teeth on each gear wheel. Calculate the speed ratio of two gears by dividing the angular velocity of the output gear (represented numerically by the number of teeth) by the angular velocity of the input gear (represented numerically by the number of teeth).

Speed Ratio Example

Say you have an input gear with 10 teeth and an output gear with 20 teeth. You find the speed ratio by working out:

$$20/10 = 2$$

This pair of gears has a speed ratio of 2, or 2/1. In other words, the input gear revolves twice to make the output gear revolve once.

Calculating Speed Output

If you know the speed ratio and the speed input, you can calculate the speed output using the formula output speed = input speed ÷ speed ratio. For example, if you have a speed ratio of 3, and the input gear revolves at 180 rpm, work out:

$$180/3 = 60$$

The output speed is 60 rpm. You can reverse this formula to work out speed input if you know the speed output and the speed ratio. For example, if you have a speed ratio of 4, and the output gear revolves at 40 rpm, work out:

$40 \times 4 = 160$ The input speed is 160 rpm.

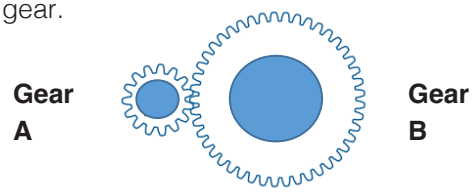
Source: https://sciencing.com/calculate-speed-ratio-7598425.htm

STATION A: UNDERSTANDING GEAR RATIOS (CONT.)

1. For the gear to the right, count the number of teeth on each gear.

   Gear A: _____

   Gear B: _____



2. For gears A and B above, gear A is the driver or input gear and gear B is driven (output gear).

   The teeth ratio if found by writing the number of teeth of the driver gear to the number of teeth of the driven gear. What is the teeth ratio for the gears above? _____

3. If Gear A were to rotate 3 times, how many times would gear B rotate? _____

4. If Gear B were to rotate 3 times, how many times would gear A rotate? _____

5. The gear ratio can be calculated by dividing the number of teeth of the driven/output by the number of teeth of the driver/input gear.

   What is the gear ratio for the gears above? _____

6. Gear ratio is also called _____

   and uses the units _____.

7. What do you notice is the mathematical relationship between the teeth ratio and the gear/speed ratio?

   _____

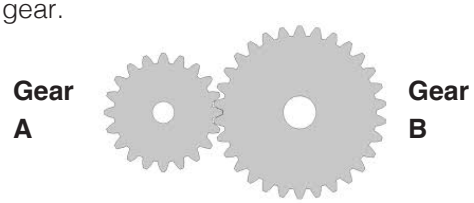
8. If gear A above rotates with an input speed of 210 rpm, what is the output speed of gear B? Remember that the speed ratio is needed to make this calculation.

9. Which gear is rotating at a faster speed—gear A or gear B?

10. For the gear to the right, count the number of teeth on each gear.

    Gear A: _____

    Gear B: _____



STATION A: UNDERSTANDING GEAR RATIOS (CONT.)

11. For gears A and B above, gear A is the driver or input gear and gear B is driven (output gear).

The **teeth ratio** if found by writing the number of teeth of the driver gear to the number of teeth of the driven gear. What is the teeth ratio for the gears above?

12. If Gear A were to rotate 3 times, how many times would gear B rotate? _____

13. If Gear B were to rotate 4 times, how many times would gear A rotate? _____

14. The **gear ratio** can be calculated by dividing the number of teeth of the driven/output by the number of teeth of the driver/input gear.

What is the gear ratio for the gears above? _____

15. If gear A above rotates with an input speed of 900 rpm, what is the output speed of gear B?

Remember that the speed ratio is needed to make this calculation.

VIDEO NOTES

Video Topic: _____

As you watch the video, write down your ideas and impressions and some facts or ideas you heard.

Things I learned:

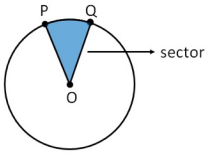
Questions I have:

Important vocabulary and other thoughts:

Opinions I have:

STATION C: CALCULATING ARC LENGTH

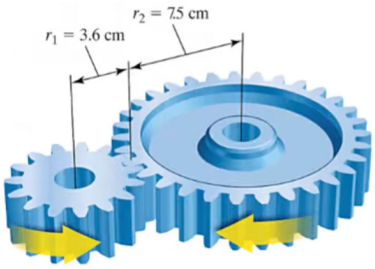
When two gears are connected and one is turning the other, you can calculate the degrees of rotation one gear moves in relation to the other. In order to make this calculation, the radius of each gear must be known. The distance from point P to Q is called the arc length of the shaded sector below



$$\frac{s}{\theta} = r$$

The trigonometric relationship between the arc length (s) is the product of the radius (r) and the central angle of the sector θ in radians. The conversion factor between degrees and radians is needed here—multiply the degrees by π divided by 180°.

Use the information labeled on the gears below to calculate how many degrees the larger gear will rotate, if the smaller/input gear rotates 300°.



First convert the central angle of 300° into radians:

$$300^\circ \times \frac{\pi}{180} = \frac{\pi}{180}\pi \text{ radians}$$

Next, it is important to note that we are looking for the arc length of the smaller gear with a radius of 3.6 cm.

Solve for the arc length by multiplying the radius by the central angle in radians.

$$\text{arc length (s)} = 3.6 \times \frac{\pi}{180}\pi = 6\pi$$

*Here it is important to note that the arc length (s) is the same for the two gears.

Now calculate the degrees of rotation of the larger gear. Once again, apply the formula: the arc length (s) is the product of the radius (r) and the central angle of the sector θ in radians.

$$6\pi = 7.5\,\theta$$

$$\frac{6\pi}{7.5} = \theta$$

$$2.5 \text{ radians} = \theta$$

Lastly, the final conversion is to convert the angle radians into degrees by multiplying the radians by 180 divided by π.

$$2.5 \times \frac{180}{\pi} = 143.2^\circ$$

The larger gear rotates 143.2° as the smaller gear rotates 300°.

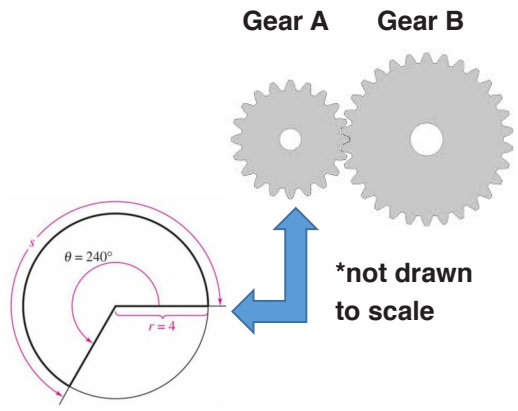
- Use the following as a resource to better understand arc length calculations. https://www.ck12.org/book/ck-12-college-precalculus/section/5.4/

STATION C: CALCULATING ARC LENGTH (CONT.)

Your Turn:

Gear A has a radius of 4 inches as labeled below and rotated 240°. Gear B has a radius of 6 inches. Find the arc length and how many degrees gear B rotated.

Be sure to show and label all of your work.



In the space below, create your own problem similar to the one above. Then switch papers with a partner and try to solve each other's problem.

Station A: Understanding Gear Ratios Answer Key

1. Gear A: 15 teeth, Gear B: 4 teeth
2. Teeth ratio: 15/45 = 1/3
3. 1 time
4. 45 times
5. 3/1
6. Gear speed, rotations per minute (rpm)
7. They are reciprocal.
8. 210/3 = 70 rpm
9. Gear A
10. Gear A: 20 teeth, Gear B: 30 teeth
11. Teeth ratio: 20/30 = 2/3
12. 2 time
13. 6 times
14. 3/2
15. 900/1.5 = 600 rpm

Station C: Calculating Arc Length Answer Key

Your Turn:

$240^\circ = \frac{4}{3}\pi$ radians

Arc length $= 4(\frac{4}{3}\pi) = 5.3\pi$

$5.3\pi = 6\,\theta$

$\theta = 2.78$ radians

2.78 radians $= 159.3^\circ$